

Choose the correct answer: (25 points)

1- Find a new representation of the equation of $2x^2 - xy + 2y^2 - 30 = 0$ after rotating through an angle $\theta = 45^\circ$. (a) $x^2 + y^2 = 9$ (b) $x^2 + y^2 - 4x = 30$ (c) $\frac{x^2}{20} - \frac{y^2}{12} = 1$ (d) $\frac{x^2}{20} + \frac{y^2}{12} = 1$

2- Find the distance between the given lines $4x - 3y = 12$ and $4x - 3y = -8$.

- (a) 3 (b) 4 (c) 5 (d) 6

3- A line through $(-5, 2)$ and $(1, -4)$ is perpendicular the line through $(x, -7)$ and $(8, 7)$. Find x

- (a) -4 (b) -5 (c) -6 (d) -19/3

4- Find the equation of the perpendicular bisector of the line joining $(5, 0)$ and $(-7, 3)$.

- (a) $8x + 2y = -11$ (b) $8x - 2y = -11$ (c) $8x - y = -11$ (d) $8x + y = -11$

5- The equation of the line through $(-2, 6)$ with the x -intercept half the y -intercept is

- (a) $x - y = 6$ (b) $2x + 2y = -2$ (c) $3x - y = -2$ (d) $2x + y = 2$

6- Find the coordinates of the point $(2, 4)$ with respect to the translated axis with origin at $(1, 3)$.

- (a) $(1, -1)$ (b) $(1, 1)$ (c) $(-1, -1)$ (d) $(-1, 1)$

7- Determine the equation of the circle whose center is at $(4, 5)$ and tangent to the circle whose equation is $x^2 + y^2 + 4x + 6y - 23 = 0$.

- (a) $x^2 + y^2 - 8x + 10y - 25 = 0$ (b) $x^2 + y^2 + 8x - 10y + 25 = 0$
(c) $x^2 + y^2 - 8x - 10y + 25 = 0$ (d) $x^2 + y^2 - 8x - 10y - 25 = 0$

8- Find the equation of the circle with center on $x + y = 4$ and $5x + 2y = -1$ and having a radius of 3.

- (a) $x^2 + y^2 + 6x - 16y + 64 = 0$ (b) $x^2 + y^2 + 8x - 14y + 25 = 0$
(c) $x^2 + y^2 + 6x - 14y + 49 = 0$ (d) $x^2 + y^2 + 6x - 14y + 36 = 0$

9- Find the equation of the circle which touches both the axis and having a radius of 5.

- (a) $x^2 + y^2 + 10x + 10y - 25 = 0$ (b) $x^2 + y^2 - 10x - 10y + 25 = 0$
(c) $x^2 + y^2 - 10x + 10y + 50 = 0$ (d) $x^2 + y^2 + 10x - 10y + 15 = 0$

10- Determine the equation of the circle whose radius is 5, center on the line $x = 2$ and tangent to the line $3x - 4y + 11 = 0$.

- (a) $(x - 2)^2 + (y - 2)^2 = 5$ (b) $(x - 2)^2 + (y + 2)^2 = 5$

- (c) $(x - 2)^2 + (y - 2)^2 = 25$ (d) $(x - 2)^2 + (y + 2)^2 = 25$

11- The focus of the parabola $y^2 = 16x$ is at

- (a) $(4, 0)$ (b) $(0, 4)$ (c) $(3, 0)$ (d) $(0, 3)$

12- The equation of the directrix of the parabola $(y - 2)^2 = 8(x - 1)$ is

- (a) $x = 3$ (b) $x = -3$ (c) $y = 3$ (d) $y = -3$

13- Given the equation of a parabola $3x + 2y^2 - 4y + 7 = 0$. Locate its vertex.

- (a) $(5/3, 1)$ (b) $(5/3, -1)$ (c) $(-5/3, -1)$ (d) $(-5/3, 1)$

14- In the equation $y = -x^2 + x + 1$, where is the curve facing?

- (a) Upward (b) Facing left (c) Facing right (d) Downward

15- Find the equation of the axis of symmetry of the parabola $y = 2x^2 - 7x + 5$.

- (a) $7x + 4 = 0$ (b) $4x + 7 = 0$ (c) $4x - 7 = 0$ (d) $x - 2 = 0$

16- A parabola has its focus at $(7, -4)$ and directrix $y = 2$. Find its equation.

- (a) $x^2 + 12y - 14x + 61 = 0$ (b) $x^2 - 14y + 12x + 61 = 0$

- (c) $x^2 + 14y - 12x + 61 = 0$ (d) None of these.

17- The lengths of the major and minor axes of an ellipse are 10 and 8, respectively. Find the distance between the foci.

- (a) 3 (b) 4 (c) 5 (d) 6

18- The equation $25x^2 + 16y^2 - 150x + 128y + 81 = 0$ has its center at?

- (a) $(3, -4)$ (b) $(3, 4)$ (c) $(4, -3)$ (d) $(3, 5)$

19- An ellipse with an eccentricity of 0.65 and has one of its foci 2 units from the center. The length of the latus rectum is nearest to?

- (a) 3.5 (b) 3.8 (c) 4.2 (d) 3.2

20- Find the equation of the hyperbola with vertices $(-4, 2)$ and $(0, 2)$ and foci $(-5, 2)$ and $(1, 2)$.

- (a) $5x^2 - 4y^2 + 20x + 16y - 16 = 0$ (b) $5x^2 - 4y^2 + 20x - 16y - 16 = 0$

- (c) $5x^2 - 4y^2 - 20x + 16y + 16 = 0$ (d) $5x^2 + 4y^2 - 20x + 16y - 16 = 0$

21- Find the equation of the hyperbola through $(5/2, 3)$ and whose asymptotes are $y = \pm 2x$.

- (a) $4x^2 + y^2 + 16 = 0$ (b) $4x^2 + y^2 - 16 = 0$ (c) $x^2 - 4y^2 - 16 = 0$ (d) $4x^2 - y^2 = 16$

22- When the origin is shifted to $(-1, 2)$ by the translation of axes, the transformed equation of $x^2 + y^2 + 2x - 4y + 1 = 0$ is

- (a) $\bar{x}^2 + \bar{y}^2 = 4$ (b) $\bar{x}^2 + \bar{y}^2 = 16$ (c) $\bar{x}^2 + \bar{y}^2 + 2\bar{x} = 4$ (d) $\bar{x}^2 + \bar{y}^2 - 2\bar{x} = 16$

23- The angle of rotation of axes to eliminate the xy term of the equation $xy - y^2 - 3y + 4 = 0$ is

- (a) $\theta = \frac{\pi}{4}$ (b) $\theta = \frac{\pi}{2}$ (c) $\theta = \frac{\pi}{3}$ (d) $\theta = \frac{\pi}{8}$

24- The point of translation of axes to eliminate the constant and the y term in the equation $y^2 + 8x + 4y - 2 = 0$ is

- (a) $(3, -2)$ (b) $(3, -2/3)$ (c) $(3/4, -2)$ (d) $(2/3, -4)$

25- If the axes are rotated through an angle $\theta = \frac{\pi}{4}$, the coordinates of $(2\sqrt{2}, -3/\sqrt{2})$ in the new system are

- (a) $(3\sqrt{3}, -5)$ (b) $(-1, -5)$ (c) $(5\sqrt{3}, -7)$ (d) $(7, \sqrt{3})$

Best wishes (Prof. Dr. Hany Atia)

Choose the correct answer

26. If $(1+i)(x+2y) - (3-i)(x-y) = 8+3i$. Then $x, y =$

(a) $\frac{7}{12}, \frac{11}{6}$

(b) $\frac{7}{12}, \frac{1}{6}$

(c) $\frac{5}{12}, \frac{3}{6}$

(d) Not given

27. The solution of $z^4 = 16$ is:

(a) $2, 2i, -2i$

(b) $2, 2i$

(c) $-2, -2i$

(d) Not given

28. By using the complex numbers compute $\sin \theta^4$:

(a) $\cos 4\theta - \cos 2\theta + \frac{3}{4}$

(b) $\frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 2\theta + \frac{3}{8}$

(c) $\frac{1}{4}\cos 4\theta - \cos 2\theta + 2$

(d) Not given

29. For $z = i + \sqrt{3}$, r and θ are:

(a) 4, 60

(b) 2, 30

(c) 1, 135

(d) Not given

30. The fourth term in $(x-2y)^5$ is:

(a) $70x^3y^3$

(b) $160x^3y^3$

(c) $-160x^3y^3$

(d) Not given

31. If $T_5 = T_6$ in $(8x^2 + \frac{1}{x})^9$. Then, $x =$

(a) 2

(b) 5

(c) 0.5

(d) Not given

(32-33) in $(2x - \frac{3}{x})^{11}$, find the following

32. $\frac{T_7}{T_5} =$

(a) $\frac{90}{23x^4}$

(b) $\frac{63}{20x^4}$

(c) $\frac{22}{3x^4}$

(d) Not given

33. The middle term is:

(a) 6

(b) 6, 7

(c) 6, 5

(d) Not given

(34-36) By using the determinant, find the following

$$2x + y - z = 4, \quad 5x - 4z - 9 = 0, \quad 3x + 4y + z = 6$$

34. $\Delta =$

(a) 10

(b) -5

(c) 20

(d) No given

35. $\Delta_x =$

(a) 5

(b) 25

(c) 12

(d) Not given

36. $z =$

(a) 1

(b) 4

(c) 0.5

(d) Not given

(37-38) for $A = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 6 & 2 \\ 2 & 0 & 1 \end{pmatrix}$, find the following

37. $F =$

(a) $\begin{pmatrix} 2 & 1 & 0 \\ 6 & 6 & 2 \\ 2 & 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 4 & 6 & 2 \\ 0 & 6 & 2 \\ 2 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 6 & 4 & -12 \\ 0 & 4 & 0 \\ 0 & -8 & 24 \end{pmatrix}$

(d) Not given

38. $A^{-1} =$

(a) $\frac{1}{24} \begin{pmatrix} 12 & 0 & 0 \\ 2 & 4 & -7 \\ 2 & 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 6 & 0 & 0 \\ 4 & 4 & -8 \\ -12 & 0 & 24 \end{pmatrix}$

(c) $\frac{1}{24} \begin{pmatrix} 6 & 0 & 0 \\ 4 & 4 & -8 \\ -12 & 0 & 24 \end{pmatrix}$

(d) Not given

39. The matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is named as:

- (a) Identity matrix (b) diagonal matrix (c) scalar matrix (d) Not given

40. The multiplication $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} 1 & 9 & 3 \\ -3 & 7 & 6 \\ 1 & -8 & 11 \end{pmatrix}$ equals:

- (a) $\begin{pmatrix} 2 & 3 & 6 \\ 3 & 2 & 24 \\ 10 & 40 & 9 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 18 & 6 \\ -12 & 28 & 24 \\ 10 & 40 & 9 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 18 & 6 \\ -12 & 28 & 24 \\ -5 & 40 & -55 \end{pmatrix}$ (d) Not given

• True or False

1. The relation $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n}{6}(n+1)(2n+1)$ is true at $n=2$.
2. The mathematical relations are proved by using partial fraction.
3. For the matrices A and B $AB=BA$.
4. For the matrices A and B $(AB)^T = A^T B^T$.
5. Every non-singular square matrix has an inverse.
6. For the matrices A, B, and C, $A_{3 \times 2} B_{3 \times 3} = C_{3 \times 3}$.
7. For $Z=-i$, Euler representation for Z is $Z=e^{-270i}$.
8. The matrix A is called symmetric if $A=-A^T$.
9. For any two complex numbers Z_1 and Z_2 , $\text{Arg}(Z_1 Z_2) = Z_1 + Z_2$.
10. For a complex number $Z=x+iy$, then $Z\bar{Z}=x^2+y^2$.

All the best
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