



سكشن (1) جبر

By: **Shimaa Adel**

اللَّهُمَّ انْفَعْنِي
بِمَا عَلَّمْتَنِي
وَعَلَّمْنِي مَا
يَنْفَعُنِي وَزِدْنِي
عِلْمًا

تسکست (I) جبر

* الأعداد المركبة *

Imaginary number

* العدد التخيلي *

$$i = \sqrt{-1}, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1 \quad \leftarrow$$

$$Z = \textcircled{a} + i\textcircled{b}$$

عدد المركب ← Z
جزء الحقيقي ← a
جزء تخيلي ← b

* الأعداد المركبة *

صورة العادي ←
صورة التخيلي ←

$$Z_1 = a_1 + ib_1, \quad Z_2 = a_2 + ib_2$$

$$\textcircled{1} Z_1 + Z_2 = (a_1 + a_2) + i(b_1 + b_2)$$

$$\textcircled{2} Z_1 - Z_2 = (a_1 - a_2) + i(b_1 - b_2)$$

$$\textcircled{3} Z_1 Z_2 = (a_1 + ib_1)(a_2 + ib_2)$$

$$= \underline{a_1 a_2} + i a_1 b_2 + i b_1 a_2 - \underline{b_1 b_2}$$

$$= (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + b_1 a_2)$$

$$(4) \frac{z_1}{z_2} = \frac{a_1 + ib_1}{a_2 + ib_2} \times \frac{a_2 - ib_2}{a_2 - ib_2}$$

$$= \frac{a_1 a_2 + \cancel{ib_2 a_1} + \cancel{ib_1 a_2} - \cancel{i^2} b_1 b_2}{a_2^2 + b_2^2}$$

$$= \frac{a_1 a_2 + b_1 b_2 + i(b_2 a_1 + b_1 a_2)}{a_2^2 + b_2^2}$$

المقام هو العدد الحقيقي

$$|z| = \sqrt{a^2 + b^2}$$

Ex

وضع العدد المركب بالصيغة الأسية:

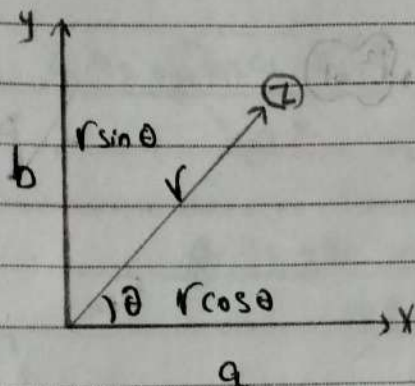
$$\begin{aligned} ① (1+i)^2 \\ (1+i)(1+i) \\ = 1+i+i+i^2 \quad (i^2 = -1) \\ = \boxed{2i} \end{aligned}$$

$$\begin{aligned} ② \frac{1+i}{1-i} \times \frac{1+i}{1+i} \\ = \frac{1+i+i+i^2}{1-i^2} \\ = \frac{2i}{2} \\ = \boxed{i} \end{aligned}$$

$$\begin{aligned} ③ \frac{(3+2i)(1-3i)}{(3+i)(1-2i)} \\ = \frac{3-9i+2i-6i^2}{3-6i+i-2i^2} = \frac{9-7i}{5-5i} \times \frac{5+5i}{5+5i} \\ = \frac{45+45i-35i-35i^2}{25-25i^2} = \frac{80+10i}{50} = \frac{8}{5} + \frac{1}{5}i \\ a = \frac{8}{5}, b = \frac{1}{5} \end{aligned}$$

* نكتب على الصورة القطبية

$$Z = a + ib$$



$$a = r \cos \theta$$

$$b = r \sin \theta$$

$$\cos \theta = \frac{a}{r}$$

$$\sin \theta = \frac{b}{r}$$

$$Z = a + ib$$

$$= r \cos \theta + i r \sin \theta$$

$$Z = r (\cos \theta + i \sin \theta)$$

$$r = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1} \frac{b}{a}$$

$$\sin \theta = \frac{b}{r}$$

$$\cos \theta = \frac{a}{r}$$

Ex

* $Z = -6i$

$a=0, b=-6 \rightarrow r = \sqrt{(-6)^2} = 6$

$\sin \theta = \frac{-6}{6} = -1 \quad \cos \theta = \frac{0}{6} = 0$

$\ominus \quad \oplus$

$\theta = -\frac{\pi}{2} \quad \theta = \frac{3\pi}{2}$

بنافه ايزاويه

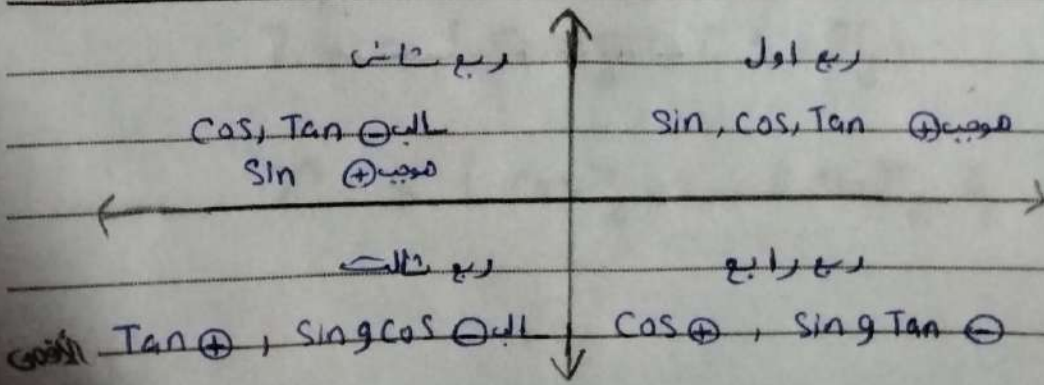
$Z = r (\cos \theta + i \sin \theta)$

$\checkmark Z = 6 (\cos(-\frac{\pi}{2}) + i \sin(-\frac{\pi}{2}))$

$\checkmark Z = 6 (\cos(\frac{\pi}{2}) - i \sin(\frac{\pi}{2}))$

$\cos \theta$ ← موجب موجب
 $\sin \theta$ ← موجب موجب

موجب



العمليات على الصورة المثلثية:

$$Z_1 = r_1 (\cos \theta_1 + i \sin \theta_1), \quad Z_2 = r_2 (\cos \theta_2 + i \sin \theta_2)$$

$$\textcircled{1} \quad Z_1 Z_2 = r_1 r_2 (\cos \theta_1 \cos \theta_2 + i \cos \theta_1 \sin \theta_2 + i \sin \theta_1 \cos \theta_2 + i^2 \sin \theta_1 \sin \theta_2)$$

$$= r_1 r_2 (\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 + i (\cos \theta_1 \sin \theta_2 + \sin \theta_1 \cos \theta_2))$$

$$= r_1 r_2 (\cos (\theta_1 + \theta_2) + i \sin (\theta_1 + \theta_2))$$

$$\textcircled{2} \quad \frac{Z_1}{Z_2} = \frac{r_1}{r_2} (\cos (\theta_1 - \theta_2) + i \sin (\theta_1 - \theta_2))$$

الحلول على الصورة التالية

① نظرية دي موافر ② نظرية دي موافر + نظرية ذات

الحيث

① نظرية دي موافر

$$Z = r(\cos \theta + i \sin \theta)^n$$

$$Z = r^n (\cos n\theta + i \sin n\theta)$$

Ex

① $(\sqrt{3} + i)^3$

$a = \sqrt{3}$, $b = 1$, $r = \sqrt{3+1} = 2$

$\sin \theta = \frac{b}{r} = \frac{1}{2}$ حيث θ حادة بين 0 و $\frac{\pi}{2}$

$$\theta = \frac{\pi}{6}$$

$$Z = 2 (\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$$

$$Z^3 = 8 (\cos \frac{\pi}{6} \cdot 3 + i \sin \frac{\pi}{6} \cdot 3)$$

② $\frac{(1+i)^4}{(1-i)^5}$

هَجَبُ كُلِّ قَوْمٍ لَوْحَةٌ ⑤

① $(1+2)^4$

$$a=1, b=1, r=\sqrt{2}$$

دریغ اول

$$\Theta = \sin^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

$$(1 + i)^4 = (\sqrt{2})^4 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)^4$$

② $(1-2)^5$

$$a=1, b=-1, r=\sqrt{2}$$

$$\sin = \frac{-1}{\sqrt{2}}, \quad \cos \theta = \frac{1}{\sqrt{2}}$$

① ②

$$\Theta = \frac{-\pi}{4} + 360^\circ, \Theta = \frac{7\pi}{4}$$

$$(1-i)^5 = (\sqrt{2})^5 \left(\cos \frac{-\pi}{4} + i \sin \frac{-\pi}{4} \right)^5$$

② is ①, and ③

10

$$\frac{(1+i)^4}{(1-i)^5} = \frac{(\sqrt{2})^4 [\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}]^4}{(\sqrt{2})^5 [\cos -\frac{\pi}{4} + i \sin -\frac{\pi}{4}]^5}$$

$$= \frac{1}{\sqrt{2}} \left(\frac{[\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}]^4}{[\cos \frac{\pi}{4} - i \sin \frac{\pi}{4}]^5} \right)$$

$$= \frac{1}{\sqrt{2}} \left[\frac{\cos \frac{\pi}{4} \cdot 4 + i \sin \frac{\pi}{4} \cdot 4}{\cos \frac{\pi}{4} \cdot 5 + i \sin \frac{\pi}{4} \cdot 5} \right]$$

$$= \frac{1}{\sqrt{2}} \left[\frac{\cos \pi + i \sin \pi}{\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}} \right]$$

في المقسم بطرح $\theta_2 - \theta_1$

$$= \frac{1}{\sqrt{2}} \left(\cos -\frac{\pi}{4} - i \sin -\frac{\pi}{4} \right)$$

Cos يتأخذ الـ 4

Sin يتغير الـ 4

$$= \frac{1}{\sqrt{2}} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$



Lecture: سكشن 2 جبر (الأعداد
المركبة)

By: Shahd Abdallah

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عِلْمًا



Subject: / جبر

موضوع الدرس: سكتش في

الاعداد المركبة

الصورة العادية

$$z = a + ib \quad |z| = \sqrt{a^2 + b^2} \quad \theta = \tan^{-1} \frac{b}{a}$$

الصورة القطبية (الفقطبية)

$$z = r(\cos \theta + i \sin \theta) \quad \theta = \tan^{-1} \frac{b}{a} \text{ or } \sin \theta = \frac{b}{r} \quad \cos \theta = \frac{a}{r}$$

$$z_1 z_2 = r_1 r_2 (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2))$$

$$|z_1 z_2| = r_1 r_2 \quad \theta = \theta_1 + \theta_2$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} (\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)) \quad \left| \frac{z_1}{z_2} \right| = \frac{r_1}{r_2}$$

صيغة ديموافر

$$z^n = [r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

Example:-

$\cos \theta, \sin \theta$ التوزيع بدلالة قوى

ذات الحديث

$$z^n = r^n (\cos n\theta + i \sin n\theta) = {}^nC_0 \cos^n \theta + {}^nC_1 \cos^{n-1} \theta (i \sin \theta) + {}^nC_2 \cos^{n-2} \theta (i \sin \theta)^2 + {}^nC_3 \cos^{n-3} \theta (i \sin \theta)^3 + \dots + {}^nC_n (i \sin \theta)^n$$

$$\sin n\theta, \cos n\theta, \tan n\theta$$

Example:-

$$(\cos \theta + i \sin \theta)^4 = \cos^4 \theta + i \sin^4 \theta + \cos^3 \theta (i \sin \theta) + \cos^2 \theta (i \sin \theta)^2 + \cos \theta (i \sin \theta)^3 + (i \sin \theta)^4$$

$$\cos^4 \theta + i \sin^4 \theta = \cos^4 \theta + 4 \cos^3 \theta (i \sin \theta) + 6 \cos^2 \theta (i \sin \theta)^2 + 4 \cos \theta (i \sin \theta)^3 + (i \sin \theta)^4$$

$$+ 4 \cos \theta (i \sin \theta)^3 + (i \sin \theta)^4$$

$$\cos^4 \theta = \cos^4 \theta - 6 \cos^2 \theta \sin^2 \theta + \sin^4 \theta$$

$$\sin^4 \theta = 4 \cos^3 \theta (i \sin \theta) - 4 \cos \theta (i \sin^3 \theta)$$

$$\tan^4 \theta = \frac{\sin^4 \theta}{\cos^4 \theta} = \frac{4 \cos^3 \theta \sin \theta - 4 \cos \theta \sin^3 \theta}{\cos^4 \theta - 6 \cos^2 \theta \sin^2 \theta + \sin^4 \theta} = \frac{\cos^4 \theta}{\cos^4 \theta} = 1$$

MITKEES

Example: - $\cos 5\theta, \sin 5\theta, \tan 5\theta$ اوجد مفكوك

$$(\cos \theta + i \sin \theta)^5 = \cos 5\theta + i \sin 5\theta = \cos^5 \theta + 5 \cos^4 \theta (i \sin \theta) + 10 \cos^3 \theta (i \sin \theta)^2 + 10 \cos^2 \theta (i \sin \theta)^3 + 5 \cos \theta (i \sin \theta)^4 + i \sin^5 \theta$$

$$\cos 5\theta = \cos^5 \theta - 10 \cos^3 \theta (\sin \theta)^2 + 5 \cos \theta (\sin^4 \theta)$$

$$\sin 5\theta = 5 \cos^4 \theta (i \sin \theta) - 10 \cos^2 \theta (i \sin^3 \theta) + i \sin^5 \theta$$

$$\tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta} = \frac{5 \cos^4 \theta (i \sin \theta) - 10 \cos^2 \theta (i \sin^3 \theta) + i \sin^5 \theta}{\cos^5 \theta - 10 \cos^3 \theta (\sin \theta)^2 + 5 \cos \theta (\sin^4 \theta)}$$

$$\tan 5\theta = \frac{5 \tan \theta - 10 \tan^3 \theta + \tan^5 \theta}{1 - 10 \tan^2 \theta + 5 \tan^4 \theta}$$

التعبير عن قوى $\cos \theta, \sin \theta$ بدلالة النسب المتكسبة

$$\cos n\theta + i \sin n\theta = z^n$$

$$\cos n\theta - i \sin n\theta = \frac{1}{z^n}$$

بشرح الطرفين

يجمع الطرفين

$$2i \sin \theta = z^n - \frac{1}{z^n}$$

$$2 \cos n\theta = z^n + \frac{1}{z^n}$$

Example: - $\cos^4 \theta$ عبر عن بدلالة النسب المتكسبة

$$(2 \cos \theta)^4 = (2^4 (\cos^4 \theta)) = (z + \frac{1}{z})^4$$

$$(2 \cos \theta)^4 = z^4 + 4C_1 z^3 (\frac{1}{z}) + 4C_2 z^2 (\frac{1}{z})^2 + 4C_3 z (\frac{1}{z})^3 + 4C_4 (\frac{1}{z})^4$$

$$(2 \cos \theta)^4 = z^4 + 4z^2 + 6 + 4\frac{1}{z^2} + \frac{1}{z^4}$$

$$(z^4 + \frac{1}{z^4}) + 4(z^2 + \frac{1}{z^2}) + 6$$

$$16 \cos^4 \theta = 2 \cos 4\theta + 8 \cos 2\theta + 6$$

Example:-

عبر عن $\sin^4 \theta$ بدلالة النسب التثلثية

$$z^4 \sin^4 \theta = (z^4 (\sin^4 \theta)) = (z - \frac{1}{z})^4$$

$$(2 \sin \theta)^4 = z^4 + 4C_1 z^3 (-\frac{1}{z}) + 4C_2 z^2 (-\frac{1}{z})^2 + 4C_3 z (-\frac{1}{z})^3 + \frac{1}{z^4}$$

$$4C_4 (-\frac{1}{z})^4$$

$$(2 \sin \theta)^4 = z^4 - 4z^2 + 6 - \frac{4}{z^2} + \frac{1}{z^4}$$

$$8 = (z^4 + \frac{1}{z^4}) - 4(z^2 + \frac{1}{z^2}) + 6$$

$$16 \sin^4 \theta = 2 \cos 4\theta - 8 \cos 2\theta + 6$$

$$z^n = r (\cos \theta + i \sin \theta)$$

$$z = \sqrt[n]{r (\cos \theta + i \sin \theta)}$$

$$= (r \cos \theta + i \sin \theta)^{\frac{1}{n}}$$

$$r^{\frac{1}{n}} (\cos \frac{\theta}{n} + i \sin \frac{\theta}{n}) \quad 2\pi k \rightarrow 0, 1, 2, 3$$

$$r^{\frac{1}{n}} (\cos \frac{2\pi k + \theta}{n} + i \sin \frac{2\pi k + \theta}{n})$$

Example:-

اوجد الجذور التكعيبي للقيمة $8i$

$$a=0 \quad b=8 \quad r=8 \quad \tan^{-1} \theta = \frac{8}{0}$$

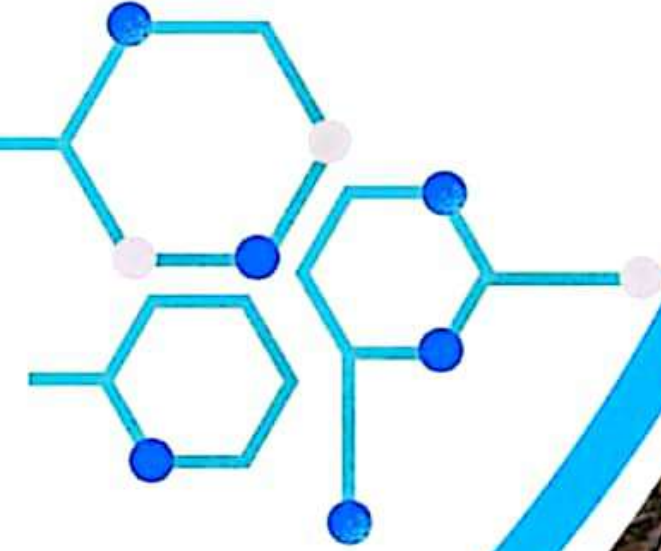
$$z^3 = 8 (\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$$

$$= 8^{\frac{1}{3}} (\cos \frac{2\pi k + \theta}{n} + i \sin \frac{2\pi k + \theta}{n})$$

$$z_0 = k=0 \rightarrow 8^{\frac{1}{3}} (\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$$

$$z_1 \quad k=1 \rightarrow 8^{\frac{1}{3}} (\cos \frac{2\pi + \pi}{3} + i \sin \frac{2\pi + \pi}{3})$$

$$z_2 \quad k=2 \rightarrow 8^{\frac{1}{3}} (\cos \frac{4\pi + \pi}{3} + i \sin \frac{4\pi + \pi}{3})$$



Lecture: (7,8,9) سكشن ثالث جبر

By: Kholoud Nader

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طريقة التكمال المربع

$$x^2 - 6x + y^2 - 2y = 15$$

$$(x^2 - 6x + 9) + (y^2 - 2y + 1) = 15$$

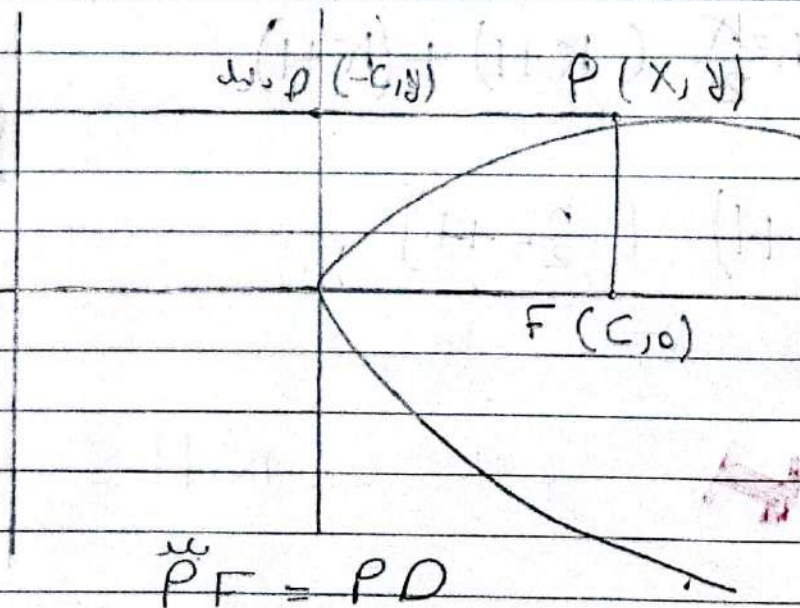
$$(x^2 - 6x + 9) + (y^2 - 2y + 1) = 25$$

$$(x - 3)^2 + (y - 1)^2 = r^2 = 25$$

$$\therefore \text{مركز } (c) = (h, k) = (3, 1)$$

$$\therefore r = 5$$

note: أي معادلة من الدرجة الثانية هو قطع مكافئ



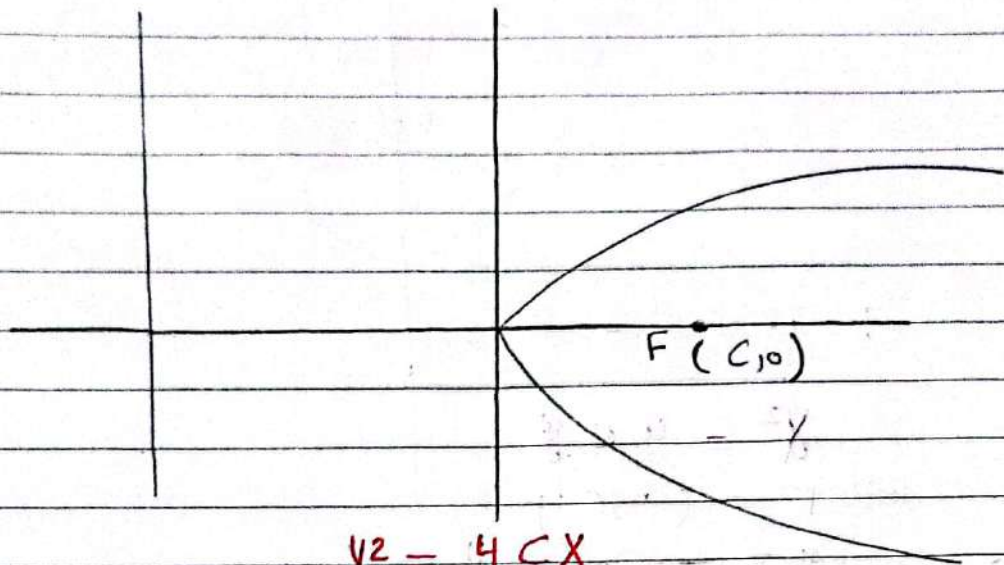
من ربع العرفين $\sqrt{(x-c)^2 + (y-0)^2} = \sqrt{(x+c)^2 + (y-y)^2} = \sqrt{(x+c)^2}$

$$(x-c)^2 + y^2 = (x+c)^2$$

$$\cancel{x^2} + \cancel{c^2} - 2cx + y^2 = \cancel{x^2} + \cancel{c^2} + 2cx$$

$$y^2 - 2cx = 2cx$$

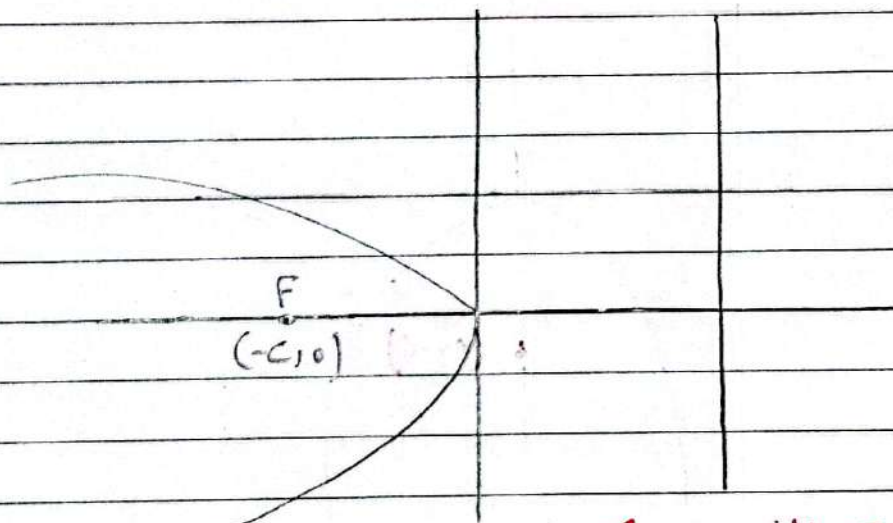
$$\Rightarrow y^2 = 4cx$$



$$y^2 = 4CX$$

$$F = (C, 0)$$

$$X = C$$



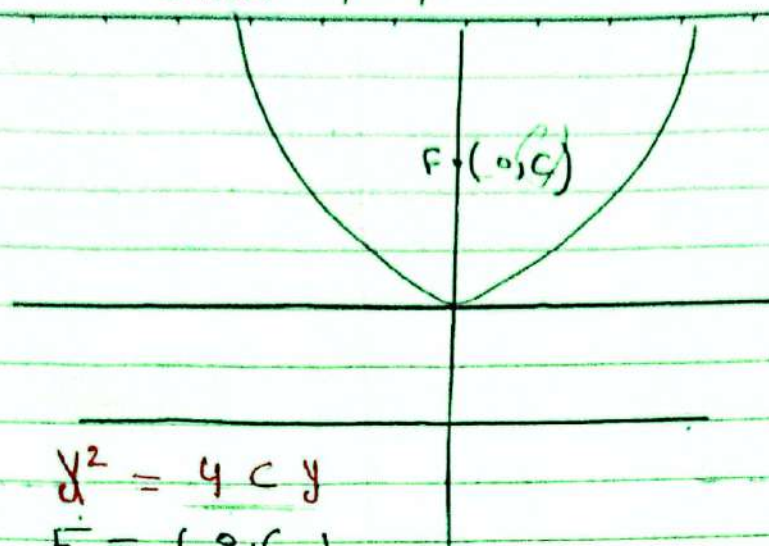
$$y^2 = -4CX$$

$$F = (-C, 0)$$

$$X = -C$$

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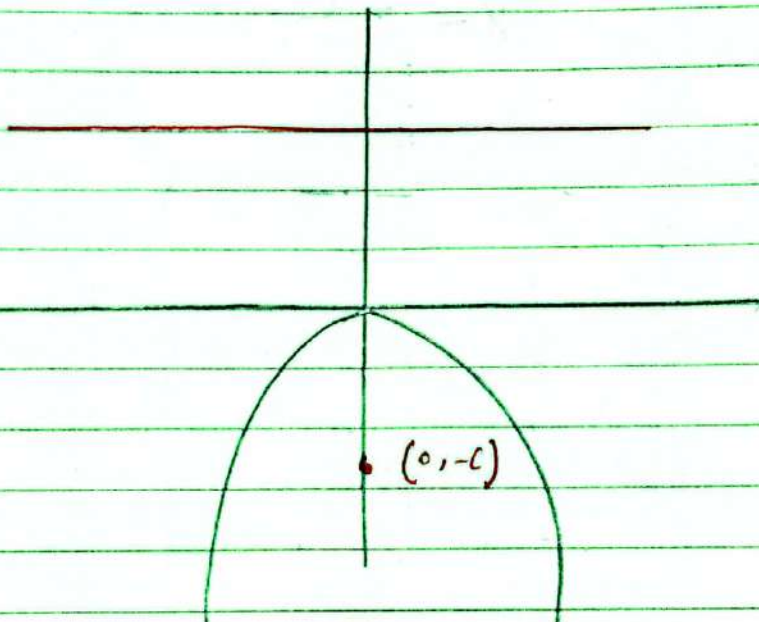
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$$x^2 = 4cy$$

$$F = (0, c)$$

$$y = -c$$



$$x^2 = -4cy$$

$$F = (0, -c)$$

$$y = c$$

أوجد معادلة قطع المكافئ الذي رأسه (0,0) ، $F = (3,0)$

$$y^2 = 4cx$$

$$c = 3$$

$$y^2 = 12x$$

$$x = -3$$

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اوجد معادلة القطع المكافئ الذي رأسه (0,0) $F(0, -2)$

$$x^2 = -4cy$$

$$c = 2$$

$$\therefore x^2 = -8y$$

اوجد معادلة القطع المكافئ الذي رأسه (0,0) $F(-\sqrt{3}, 0)$

$$y^2 = -4cx$$

$$c = \sqrt{3}$$

$$x = -\sqrt{3}$$

$$\therefore y^2 = -4\sqrt{3}x$$

اوجد معادلة القطع المكافئ الذي رأسه (0,0) $F(0, 3)$

$$x^2 = 4cy$$

$$c = 3$$

$$\therefore x^2 = 12y$$

اوجد معادلة القطع المكافئ الذي رأسه (0,0) $F(-5, 0)$

$$y^2 = -4cx$$

$$c = 5$$

$$\therefore y^2 = -20x$$

$$x = -5$$

اوجد معادلة القطع المكافئ الذي رأسه (0,0) $F(3, 0)$

$$y^2 = 4cx$$

$$x = -3$$

Hawar

$$c = 3$$

$$\therefore y^2 = 12cx$$

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$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 \times 3 & 0+3 \\ -2+8 & 0+4 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 6 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 3 \\ -1 & 3 & 0 \\ 4 & 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 1 \\ 1 & 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3+3 & 4+0+9 & 2+1+9 \\ -1+9+0 & -2+0+0 & 1+3+0 \\ 4+6-1 & 8+0-3 & 4+2-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 13 & 12 \\ 8 & -2 & 4 \\ 9 & 5 & 3 \end{bmatrix}$$

Nawar