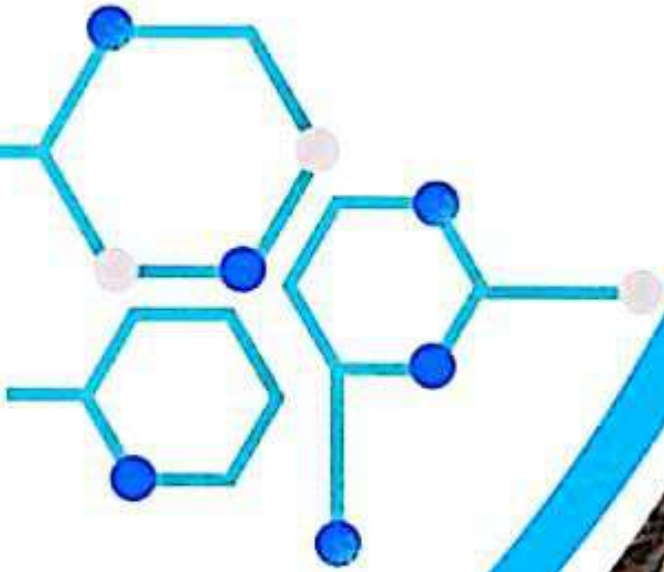




Lecture: ملخص الهندسه التحليليه

By: Anas Mohamed

اللَّهُمَّ انْفَعْنِي
بِمَا عَلَّمْتَنِي
وَعَلَّمْنِي مَا
يَنْفَعُنِي وَزِدْنِي
عِلْمًا

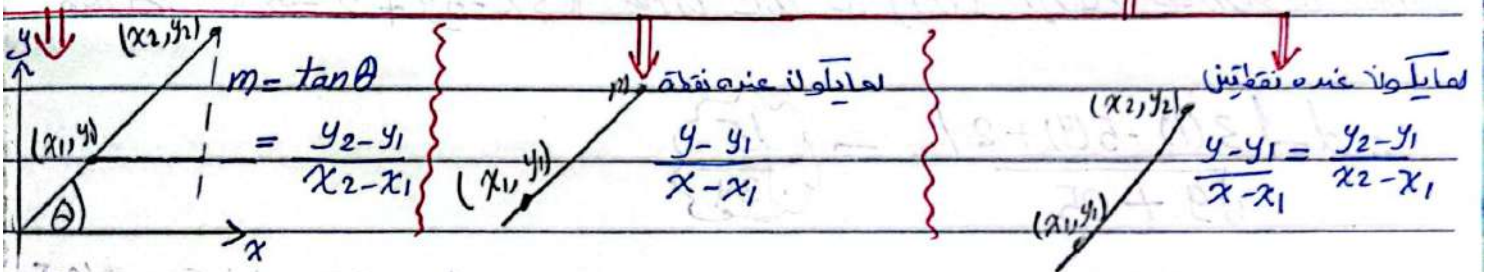
«ماخص: هندسة التحليلية»

«النبى محمد»

«النبى محمد»

نول حاجة كديا باشا ما جعنا أنواع الأشكال الهندسية

وهو الخط المستقيم



نقطة midpoint:

$$\frac{y - y_1}{x - x_1} = m \quad \text{[4] (نقطة في)} \quad \text{ميل}$$

نقطة midpoint

لو كان معاك نقطتين وهو عاوز العمود:

1) بتجيب midpoint ونقطة المنتصف

2) بتجيب slope ميل من خلال نقطتين أو أى كان

$$m_1 = \frac{1}{m_2}$$

عكس عكس

3) بتجيب الميل لو هو

$$m_1 = m_2 \text{ موازى}$$

$$Ax + By + C_1 = 0 \text{ موازى}$$

$$Bx - Ay + C_2 = 0 \text{ عمودى}$$

$$Ax + By + C = 0 \text{ general form}$$

لو معاك

وهو عاوز m

$$m = \frac{-A}{B}$$

$$m = \frac{B}{A}$$

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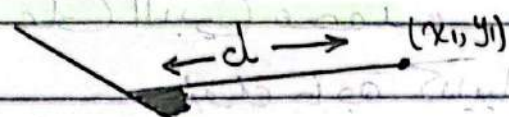
Ex: Find an eqn of the Line: (a) Parallel (b) perpendicular to $3x + 2y - 5 = 0$ and contain the point $(3, 1)$

The solves:

$$(a) 3x + 2y + C_1 = 0 \Rightarrow \text{from } (3, 1) \Rightarrow 3(3) + 2(1) + C_1 = 0 \therefore C_1 = -11$$

$$(b) 2x - 3y + C_2 = 0 \Rightarrow \text{from } (3, 1) \Rightarrow 2(3) - 3(1) + C_2 = 0 \therefore C_2 = -3$$

$$\therefore Ax + By + C = 0$$



(d) $d = \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}$

The distance point (1, 4) to the line $3x - 5y + 2 = 0$ is

$$\therefore d = \frac{|3(1) - 5(4) + 2|}{\sqrt{9 + 25}} = \frac{15}{\sqrt{34}}$$

المسافة من النقطة إلى الخط

① أ ب (2, 2), $m = 1$ $\therefore \frac{y-2}{x-2} = 1 \therefore x-2 = y-2 \Rightarrow x-y=0$
 $\therefore \boxed{x=y}$

② أ ب (4, -2), $m = 0$

$$\therefore \frac{y+2}{x-4} = 0 \therefore y+2=0 \Rightarrow \boxed{y=-2}$$

③ أ ب (2, 5), no slope

$$\Rightarrow \boxed{x=2}$$

④ a (1, 4), b (3, 0), c (-1, -2)

Find the equation chord of the circle $x^2 + y^2 = 25$ which joins (-3, 4), (5, 0)

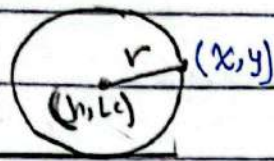
The slope

$$\therefore \frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \therefore \frac{y-4}{x+3} = \frac{0-4}{5+3}$$

$$\therefore \frac{y-4}{x+3} = -\frac{1}{2} \Rightarrow 4x-12 = 8y-32 \Rightarrow 4x-8y = 12-32 \Rightarrow 4x-8y = -20 \Rightarrow 4x+8y+20=0 \Rightarrow \boxed{x+2y+5=0}$$

ثاني حاجة بقى يصعب وهي الدائرة:

The circle:



$$\text{مسافة} = r = \sqrt{(x-h)^2 + (y-k)^2}$$

$$\text{صيغة قياسية} \Rightarrow (x-h)^2 + (y-k)^2 = r^2$$

$$\text{General form} \Rightarrow x^2 + y^2 + Ax + By + C = 0$$

$$\text{Ex: } 2x^2 + 2y^2 - 2x + 6y - 3 = 0 \quad \div 2$$

$$x^2 + y^2 - x + 3y - \frac{3}{2} = 0$$

$$\text{نقوم بعمل مكمل مربع} \Rightarrow$$

$$\therefore (x - \frac{1}{2})^2 - \frac{1}{4} + (y + \frac{3}{2})^2 + \frac{9}{4} - \frac{3}{2} = 0$$

في هذا زنت عارفي نعمل أكمل مربع:

$$① \text{ مثلاً عندك } x^2 - x \text{ فأخذنا } x^2 - x \text{ أكمل مربع}$$

فأول حاجة بتأخذ x كمنفي أسالة يعني كذا

$$(x - \frac{1}{2})^2$$

② نأخذ y دائرة (الـ y) نأخذ y لو حصة الـ y رقم تقسمه على 2 تمام

$$④ \text{ نأخذ القوس } (x - \frac{1}{2})^2 \text{ ونأخذ القوس } (y + \frac{3}{2})^2$$

⑤ تربيع الرقم الذي جوا القوس ونحافظ على إشارة ما لو سالب سبوا إيجابيا

$$(x - \frac{1}{2})^2 - (\frac{1}{4}) + (y + \frac{3}{2})^2 + (\frac{9}{4})$$

وبما أننا نريد أن نحصل على x أعاد مع $\frac{1}{4}$

وأول حصة أكملت أكملت (x) وأخذت $(+)$ وأخذت $(+)$

ونأخذ أكمل مربع في (y) * نأخذ $(+)$ نأخذ $(+)$

$$(x - \frac{1}{2})^2 + (y + \frac{3}{2})^2 - \frac{1}{4} + \frac{9}{4} - \frac{3}{2} = 0 \Rightarrow (x - \frac{1}{2})^2 + (y + \frac{3}{2})^2 = 1$$

$$\therefore (x - \frac{1}{2})^2 + (y + \frac{3}{2})^2 = (1)^2 \Rightarrow (x - \frac{1}{2})^2 + (y + \frac{3}{2})^2 = 1 \quad (*)$$

$$\therefore x = \frac{1}{2} \quad y = -\frac{3}{2} \quad \text{صيغة قياسية}$$

$$\therefore \text{نقطة المركز} = (\frac{1}{2}, -\frac{3}{2})$$

center

Ex 1: find an equation of the circle through $(1, 5)$, $(-2, 3)$, $(2, -1)$
lec 2 The solve:

Chord $\Rightarrow$ ①

$$\text{slope} = \frac{5-3}{1+2} = \frac{2}{3}$$

$$\text{mid point} = \left(\frac{1-2}{2}, \frac{5+3}{2} \right) = \left(-\frac{1}{2}, 4 \right)$$

Perpendicular bisector $\Rightarrow$ ①

$$\text{Slope} = \frac{-3}{2} \quad (\text{because } (-\frac{1}{2}, 4))$$

$$\Rightarrow \frac{y-y_1}{x-x_1} = m \Rightarrow \frac{y-4}{x+\frac{1}{2}} = \frac{-3}{2}$$

$$\Rightarrow 2y-8 = -3x - \frac{3}{2}$$

$$\Rightarrow 3x+2y = \frac{13}{2} \quad \text{①}$$

Chord $\Rightarrow$ ②

$$\text{slope} = \frac{5+1}{1-2} = \frac{6}{-1} = -6$$

$$\text{mid point} = \left(\frac{3}{2}, \frac{4}{2} \right) \Rightarrow \left(\frac{3}{2}, 2 \right)$$

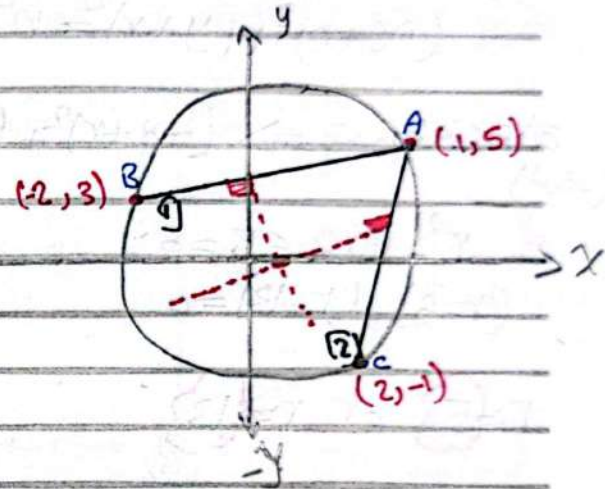
Perpendicular bisector $\Rightarrow$ ②

$$\text{slope} = \frac{1}{6} \quad (\text{because } (\frac{3}{2}, 2))$$

$$\frac{y-2}{x-\frac{3}{2}} = \frac{1}{6} \Rightarrow 6y-12 = x - \frac{3}{2} \Rightarrow -x+6y = \frac{21}{2} \quad \text{②}$$

* by solving ①, ② we get

$$\Rightarrow \text{The center of the circle } \left(\frac{9}{10}, \frac{19}{10} \right)$$



The radius is the distance between on the circle

$\Rightarrow$ The equation of the circle

$$\left(x - \frac{9}{10} \right)^2 + \left(y - \frac{19}{10} \right)^2 = \frac{962}{100}$$

$$r = \sqrt{(1+2)^2 + (3-5)^2}$$

Ex: find an eqn. of the circle tangent to both axes and containing the point $(-8, -1)$

The solve:

$$(X-h)^2 + (y-k)^2 = r^2$$

$$\because h = -r \quad k = -r$$

$$\because (X+r)^2 + (y+r)^2 = r^2$$

$$\text{أعطى النقطة } (-8, -1) \Rightarrow (-8+r)^2 + (-1+r)^2 = r^2$$

$(-8, -1)$

$$\because r^2 - 18r + 65 = 0$$

$$(r-5)(r-13) = 0$$

$$r = 5 \text{ or } r = 13$$

$$\because \text{if } r = 5$$

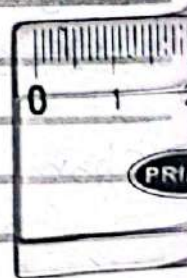
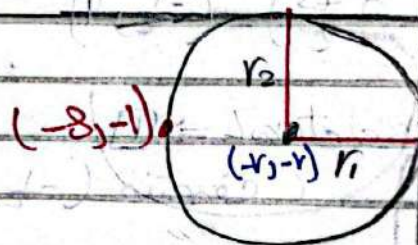
$$\because \text{center } (-5, -5)$$

$$\because (X+5)^2 + (y+5)^2 = 25 \rightarrow \textcircled{1}$$

$$\text{if } r = 13$$

$$\because \text{center } (-13, -13)$$

$$\because (X+13)^2 + (y+13)^2 = 169$$



Conic sections: القطوع المخروطية

Parabola
قطع مكافئ

Ellips
قطع ناقص

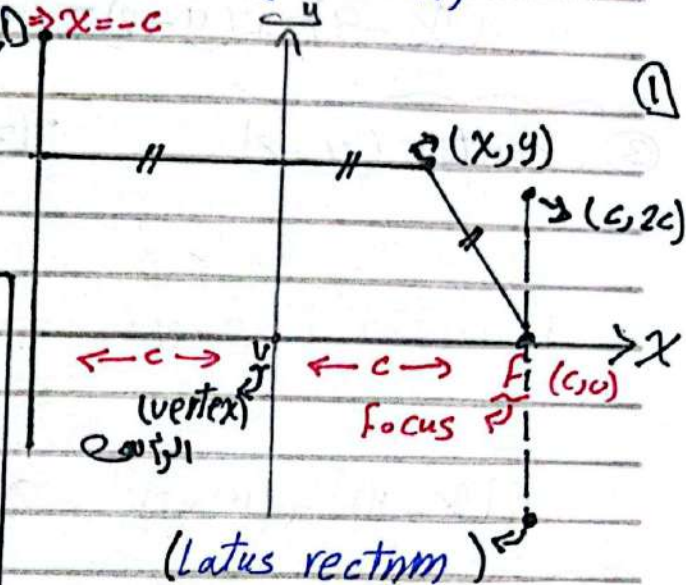
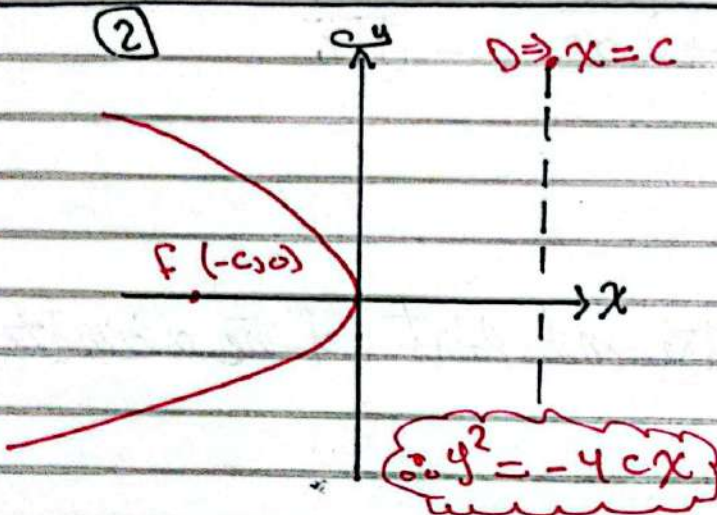
Hyperbola
قطع زائد

Parabola is the set of all points in a plane equidistant from a fixed point (focus) and a fixed line (directrix).

directrix $D \Rightarrow x = -c$
مستقيم

محور قطع $\Leftarrow$ هو محور (X)

(2)



$\Rightarrow$ (length of the latus rectum) = $4c$

From $y^2 = 4cx$

$$y^2 = 4c \times c \Rightarrow y^2 = 4c^2$$

$$\therefore y = \pm 2c$$

$$y^2 = 4cx$$

Ex: sketch and discuss

$$y^2 = 8x \Rightarrow c = 2$$

$$y^2 = 4cx \Rightarrow 4c = 8$$

$$D: x = -2$$

$$V = (0, 0)$$

$$F = (2, 0)$$

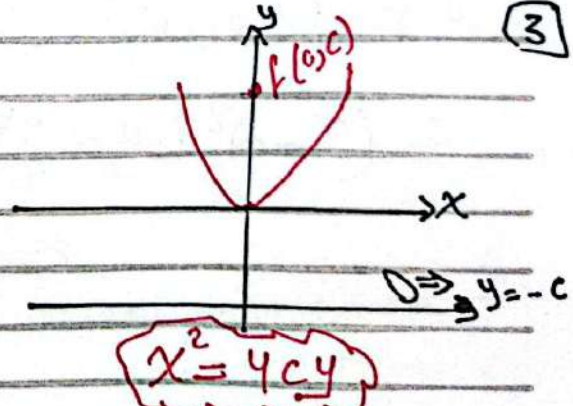
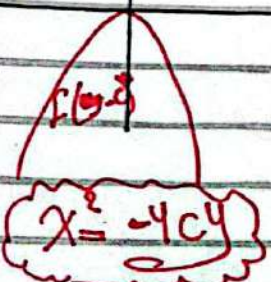
$$D: x = -2$$

$$\Rightarrow$$
 The axis of two parabola

$$\therefore y = 0$$

$$\Rightarrow$$

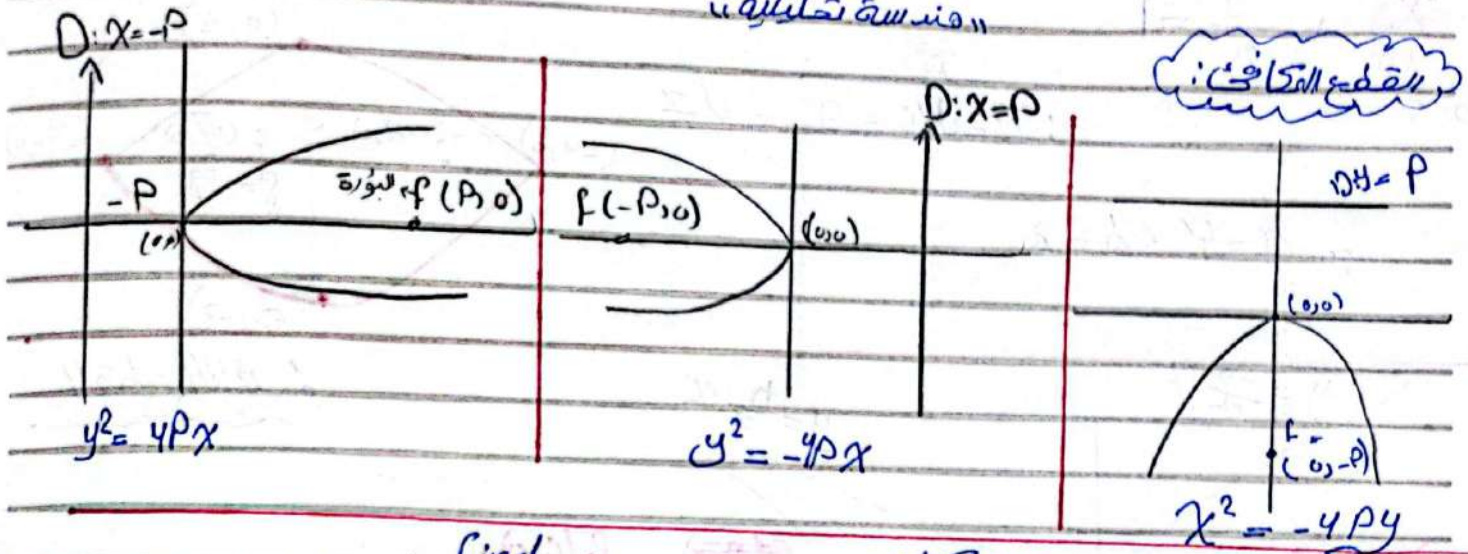
$$D \Rightarrow y = c$$



sec: 4

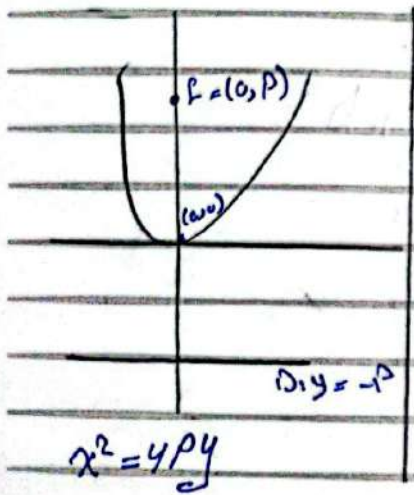
« هندسة تحليلية »

« قطع المكافئ »

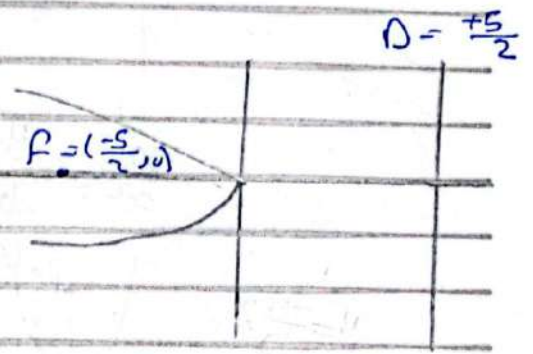


Find

« يوجد المقطع المكافئ صدارة »

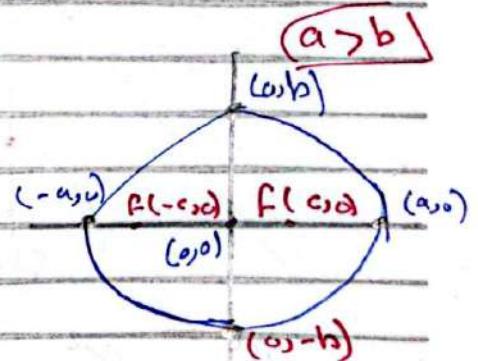
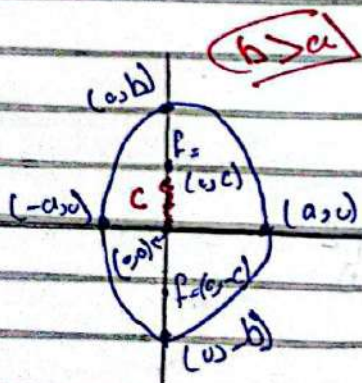


$$\begin{aligned} y^2 + 10x &= 0 \\ y^2 &= -10x \\ \therefore p &= -\frac{10}{4} = -\frac{5}{2} \end{aligned}$$



جزء المقطوع من محور x $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

« المقطع الناقص »:-



$$c^2 = a^2 - b^2$$

« الجزء الأكبر »
« الجزء الأصغر »

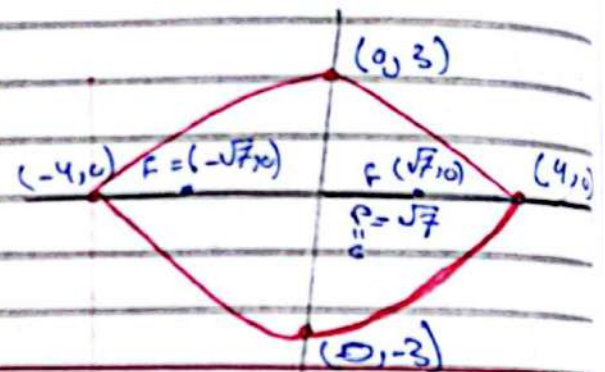
محمد و أميرة

Ex

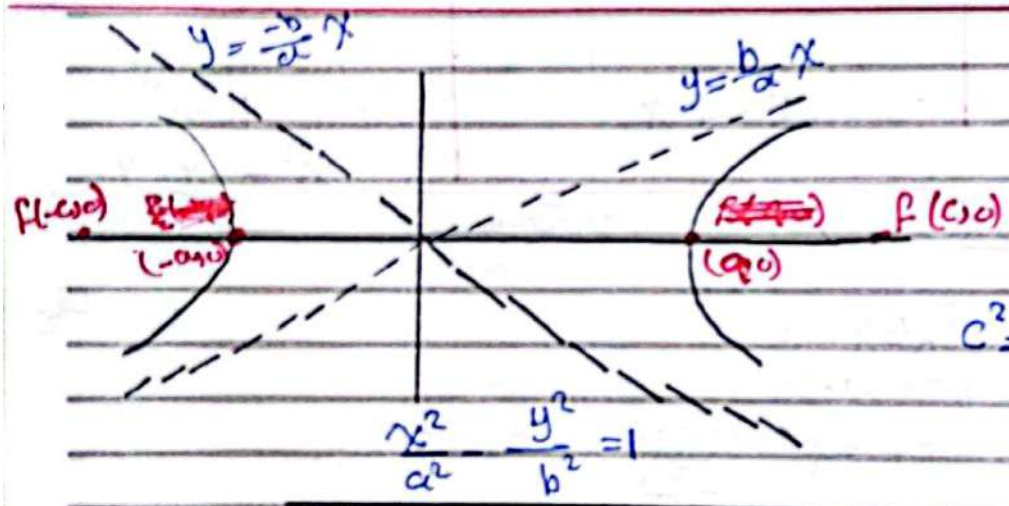
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\therefore c = \sqrt{b^2 - a^2} = \sqrt{7}$$

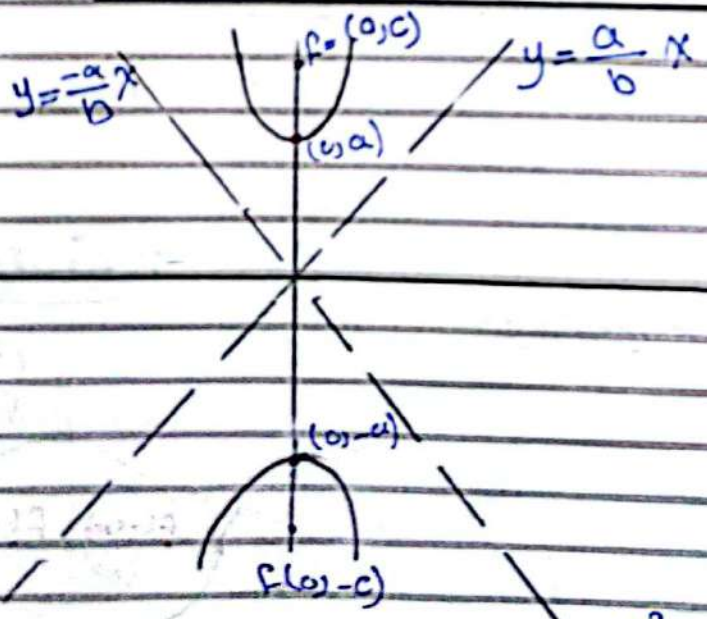
$$\therefore a=4, b=3$$



القطع الزائد



$$c^2 = a^2 + b^2$$



$$c^2 = a^2 + b^2$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



Ex: $\frac{x^2}{16} - \frac{y^2}{9} = 1$

$a = 4, b = 3$

$c = \sqrt{16 + 9} = \sqrt{25} = 5$

